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ABSTRACT

In dynamical theory, mathematical understanding is considered to be that of a person (or group) of a topic (or problem) in a situation or setting. This paper compares the interactions between the situations and the mathematical understandings of two students by comparing the growth in understanding within a Fibonacci sequence setting in which specific tasks were suggested and interventions made, with that of the same students in a Fibonacci setting in which only a general prompt was offered. In the former setting, the growth of understanding was characterized by jumps, indicating a collection of specific images or patterns. In the second setting, these students exhibited a continuous, non-linear pathway of understanding more governed by epistemological interest and featuring more formulated reasoning. Contains 11 references. (Author/MKR)

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Formulating the Fibonacci Sequence: Paths or Jumps in Mathematical Understanding

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FORMULATING THE FIBONACCI SEQUENCE:PATHS OR JUMPS IN MATHEMATICAL UNDERSTANDING

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In the dynamical theory of mathematical understanding (Pirie and Kieren, 1994) understanding is considered to be that of a person (or group) of a topic (or problem) in a situation or setting. In this paper we compare the interactions between the situations and the mathematical understandings of two students by comparing the growth in understanding in a Fibonacci sequence setting in which specific tasks were suggested and interventions made, with that of the same students in a Fibonacci setting in which only a general prompt was offered. In the former, the growing understanding was characterized by jumps, indicating a collection of specific images or patterns. In the second these students exhibited a continuous, non-linear pathway of understanding more governed by epistemological interests and featuring more formulated reasoning.

How Does Mathematical Understanding Grow?

There have been numerous useful ways of thinking about mathematical understanding in terms of types or levels over the past 20 years (e.g. Skemp, 1976; Herscovics and Bergeron, 1993). Under such work mathematical understanding tends to appear as an acquisition or sets of such acquisitions. Following a more phenomenological, constructivist and enactivist view of understanding (von Glasersfeld, 1987; Johnson, 1987), Kieren and Pirie over the last eight years have been building and testing a dynamical theory of the growth of mathematical understanding which views it as a non-linear, non-monotonic, process-in-action. As illustrated in the diagrams below, we observe such change in understanding in action using pathways across eight embedded levels or modes of understanding. Starting from a person's assumed primitive knowing (related to the mathematical situation in which they find themselves) their understanding, if it is not disconnected, grows through three informal modes of action (image making, image having and property noticing) and through three potential formal modes of mathematical activity (formalizing, observing and structuring) and possibly leads to a person developing new diverging mathematical ideas (inventising). The inner, informal modes of understanding are related to more local, image-related knowing and frequently involve unformulated reasoning (Reid, 1995), while the outer levels are more sophisticated and general in nature. But as we have illustrated in a number of studies, less formal understanding is fully implicated in outer, more sophisticated understanding in that students very frequently "fold back" to inner, less formal understanding action. Such folding back typically leads to a broadening of student understanding and appears to be an important if not a necessary condition in its growth.

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We are increasingly observing that understanding-in-action is best understood in terms of the inter-actions in which the person engages (e.g. Pirie and Kieren, 1992; Davis, 1994). For example, a teacher may intervene with students in the situation in an attempt to provoke them to move to more general or sophisticated understanding. Such moves are said to have *provocative intent.* Similarly, the teacher may try to have students fold back to inner less formal activity, such moves having an *invocative intent.* We have observed in a number of studies (e.g. Pirie and Kieren, 1992; Kieren, Pirie and Reid, 1994), that it is the subsequent actions of the students which determine the nature of the intervention and not the intent of the teacher.

The Nature of this Study

In a 1994 paper we reported on the understanding of two university students, Stacey and Kerry as they spent approximately an hour investigating a problem situation which grew out of a prompt which asked them to write the recursive rule defining the sequence, if they knew it or could discover it, and to "Look for patterns which relate the index n to the Fibonacci number F_n. For example, is there anything special about F_n when n is a multiple of 3, or a multiple of 4, or prime?" Using a methodology described below their (joint) understanding of the Fibonacci setting was characterized in Diagram 1.

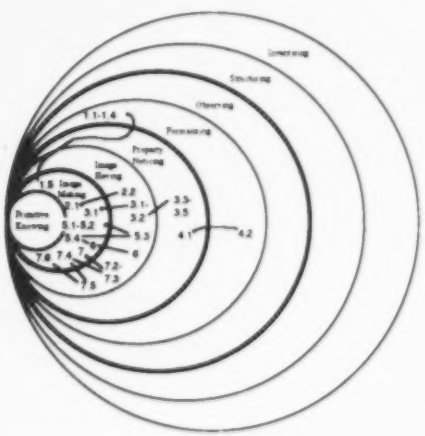

Diagram 1. The first Fibonacci session

Stacey and Kerry's growth in understanding of this Fibonacci situation seems to occur in disjoint jumps rather than as a more continuous pathway. In part this could be attributed to the prompt which in addition to asking for generalizations, gives a series of tasks to be accomplished. In addition the transcripts and mathematical activity trace of the setting reveal that the researcher had responded to requests from Stacey and Kerry with prompts of more things to do with the Fibonacci setting, which they appeared to treat as separate mathematical items. The question for us was, "Is Stacey and Kerry's growing understanding of the Fibonacci

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sequence inherently like this or should the interaction pattern with the researcher and setting be observed as an important part of Stacey and Kerry's growth in understanding?"

To study this question we provided Stacey and Kerry with an opportunity to respond to an altered Fibonacci prompt, some 15 months after their first session. In the second setting the prompt indicated: "Generalize a property of this sequence" and the researchers said nothing to the pair but simply observed their activities. Because we are attempting to observe understanding as an on-going lived activity in and with an environment, we use a number of inter-related methods both to gather and interpret the data. We term this method "brico-logical." It is a *brico*lage in that the researchers work interactively with various given materials on a piece of a more global problem. At the same time each researcher brings with them a particular *logic* of inquiry, here the Pirie/Kieren model, ideas on reasoning and proving, and the theory of enactivism (Varela, Thompson and Rosch, 1991). Each session was recorded using video tapes, transcripts and observer notes. Three different researchers, the authors, viewed the tapes and interacted and converged on possible conclusions to be drawn about understanding. Mathematical activity traces for both Fibonacci settings were developed in which major episodes of the sessions were identified and characterized. The students themselves were interviewed as to what they observed about their own thinking and researcher observations about it. These deliberations are summarized in the pathways (or jumps) on the Pirie/Kieren model.

Results and Reflections

This research is part of a multi-year study of university student mathematical understanding which itself is part of a larger eight year study of the growth in mathematical understanding in action involving students of many ages. The data gathered and interpretations developed in even these two settings represent a multi-dimensional phenomenon. This is true both because growing mathematical understanding is observed to have a recursive fractal character and because the enactive view which we are taking encourages us to consider many elements in the interaction between the students and the world they are both creating and living in "all-at-once". The report here is limited to only some of the dimensions of the situation, particularly growing understanding and patterns of reasoning as these arise for these two students.

We turn first to the "interventionist" Fibonacci setting growing understanding in which is illustrated in Diagram 1, above. The transcript and subsequent reflections of the researchers indicate that when Stacey and Kerry worked on each of the given tasks (e.g. defining the Fibonacci sequence; describing the character of every third Fibonacci number) mainly by trying a number of numerical cases and looking for a pattern. While in the case of prime indexed Fibonacci numbers the pair developed a numerically based property of the sequence (3.1-3.5 in diagram 1) and also engaged in some disjoint formalizing on the task of finding a pattern in F_{3n}, they did not develop well formulated generalizations about the Fibonacci se-

quence. In a series of interventions with *provocative intent*, the researchers then offered the pair of students a number of tasks calling for looking for inter-relationships within and between various sub-sequences of the Fibonacci sequence, hoping that the students would develop and justify more formal and sophisticated generalizations. But instead each of these suggestions invoked the students to "fold (or in this case jump) back" to working with specific numerical examples and observing a few local patterns or some numerical inter-relationships (paths 5, 6, 7 on Diagram 1). Thus while the researchers had intended that the students formulate their reasoning about this sequence by finding and explaining generalizations (which they had done in other settings), the students instead tended to do short numerical explorations and their discussion with one another remained at the level of particular numerical examples. This pattern of intervention repeated itself several times and the resulting observed understanding appears as a series of jumps.

Notice that in observing Stacey and Kerry's understanding in this setting as such a series of "jumps," we are not devaluing the knowledge of the Fibonacci sequence developed during this activity. Nor would it be appropriate to suggest that this pattern of understanding would be related to what Skemp would call instrumental understanding—Stacey and Kerry could give local justifications for their numerically based images of the Fibonacci sequence. In fact their understanding in this situation might be described as a collection of independent images and their reasoning could be characterized as exploring to seek local patterns. But we are arguing that the students' growth in understanding here co-emerges with the occasions provided in the situation, particularly the apparently discrete tasks in the initial prompt and the interventions of the researcher.

This is well illustrated when we compare the understanding diagram above with that given below. Remember that in the second setting the prompt was to generalize a property of the given Fibonacci sequence and the researchers made no further active interventions. Overall, one might characterize the growing understanding here as a non-linear growth from re-establishing an image(s) of the Fibonacci sequence to more general formalizing about this sequence. While their understanding in the first setting entailed informal reasoning with numerical examples, in the second setting it is better described as *formulating*. Stacey and Kerry re-established their image of the Fibonacci sequence (including their active re-memberence of one researcher "prohibiting" them from developing a bi-directional sequence) (1.1-1.5 on Diagram 2). They then spent the next 45 minutes elaborating and formulating that image. In particular, they focused on how one defines the "givens" for *the* Fibonacci sequence. At 2.5 Stacey noted a property of *the* Fibonacci sequence: that the rule limits the number of givens needed to two. After characterizing their image of the sequence as possibly being members of the natural numbers or integers (4-6), at Stacey's urging they escaped the boundaries of *the* Fibonacci sequence and folded back (7.1) to exploring and formalizing a bi-directional Fibonacci sequence (7.2-7.3). This latter formalizing activity was clearly connected to their previous thinking; for example, Kerry specifically related in his formalizing about the bi-directional sequence to the discussion they had earlier about the members of the sequence being integers (7.3).

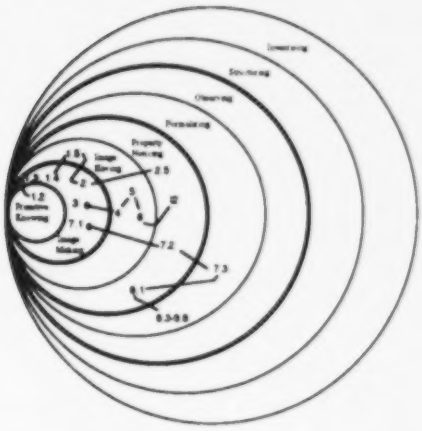

Diagram 2. The second Fibonacci session

Using the bi-directional Fibonacci sequence as a stimulus, Kerry noticed a specific property—one can define the sequence using terms which are separated (F_1=1 and F_{-1} = 1) (8.1, 8.2). They spent the rest of the time formalizing, carefully formulating and re-formulating and justifying a new general definition of *the* Fibonacci sequence (8.3-8.8) as illustrated by this interchange:

Kerry:	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. x_{11}, so, x_1=1 and x_{11}=89. Tada! That's the Fibonacci sequence. I defined it.
Stacey:	Do you always have to start with x_1 = 1?
Kerry:	No, don't have to- I'll get a new x. x_9=34, so- What do we have here? x_{11}=89, x_9=34, that's your Fibonacci sequence. You can not come up with any other sequence if you follow those rules.

It would be easy to conclude that the fact that the researchers made no interventions allowed Stacey and Kerry's understanding to grow in a particular way, but that would be an over-simplification. The researchers' mere presence likely provoked the continuous and more formulated and general formalizing which these two students exhibited. But they also felt constraints that had arisen in the first session. In that session a researcher had noted that *the* Fibonacci sequence had no "zeroth" terms nor any "negative" terms. Although the second setting occurred over a year later, both Stacey and Kerry re-membered and re-constructed this dialogue and this remembrance acted as a constraint on much of their activity, as did their remembered dissatisfaction with their more disjoint previous activity with the problem. Under these constraints they focused on what they called *the* Fibonacci sequence. Their understanding activities can be seen as influenced by what Sierpinska (1994) identifies as epistemological concerns. In fact, their well formulated formalizing (8.1-8.8), and even their whole pathway of understanding centers around generalizing and formalizing conditions underlying *the* Fibonacci sequence.

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In the continuous non-linear pathway of growth of understanding in setting two, Stacey and Kerry's reasoning appeared to be governed by a need to explain rather than a need to explore (Reid, 1995). Again we are not saying that such a pathway is evidence of better or more productive understanding than that in the first setting, but that this growing understanding coemerged with the features of the prompt, the (non) actions of the researchers and with the developing epistemological concerns of the students. The students themselves did sense a difference between their understandings in the two settings:

Kerry: Yeah. Cause we- When we walked away from it [setting one] neither one of us felt we'd really climbed a mountain or conquered anything.

Stacey: No.

Kerry: But now, I'm quite happy with it now. The Fibonacci sequence is allowed back in my life.

Stacey: Yeah [laughter].

References

Davis, B. (1994) *Listening to Reason*. An unpublished doctoral thesis at the University of Alberta. Edmonton: University of Alberta, Faculty of Graduate Studies, Department of Secondary Education.

Herscovics, N. and J. Bergeron (1993) An epistemological study of the child's construction of early number. *Alberta Journal of Educational Research* 39 (1) pp. 23-39.

Johnson, M. (1987). *The body in the mind: The bodily basis of meaning, imagination, and reason*. Chicago: University of Chicago Press.

Kieren, T., Pirie, S. & Reid, D. (1994). Mathematical understanding: Always under construction. In J. da Ponte & J. Matos, (Eds.), *Proceedings of the Seventeenth Annual Conference of the International Group for the Psychology of Mathematics Education*, (Vol. 4, pp. 49-56). Lisbon.

Pirie S. and T. Kieren (1992) Creating Constructivist Environments: Constructing Creative mathematics. *Educational Studies in Mathematics*. 23(5) pp. 505-528.

Pirie S. and T. Kieren (1994) Growth in mathematical understanding: how can we characterise it and how can we represent it? *Educational Studies in Mathematics* 26: pp. 165-190.

Reid, D. (1995) *The need to prove*. Unpublished doctoral dissertation, University of Alberta Edmonton: University of Alberta, Faculty of Graduate Studies, Department of Secondary Education.

Sierpinska, A. (1994) *Understanding in mathematics*. London: Falmer.

Skemp, R. (1976) Relational Understanding and Instrumental Understanding. *Mathematics teaching*, No. 77, pp. 20-26.

Varela, F., Thompson, E., & Rosch, E. (1991). *The embodied mind: Cognitive science and human experience*. Cambridge MA: MIT Press.

von Glasersfeld, E. (1987) Learning as a constructive activity. In C. Janvier (Ed.), *Problems of representation in the learning and teaching of mathematics*. (pp. 3-17). Hillsdale NJ: Lawrence Erlbaum Associates.